**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Soln. :**

We have a normal distribution with *μ* = 45 and *σ* = 8.0.

Let, X be the amount of time it takes to complete the repair on a customer's car.

To finish in one hour you must have X ≤ 50

to find : P(X > 50).

P(X > 50) = 1 - P(X ≤ 50).

Z = (X - 45)/8.0

P(X ≤ 50) = P(Z ≤ (50 - 45)/8.0)

= P(Z ≤ 0.625)

=73.4%

Probability that service manager will not meet his demand will be = 100-73.4 = 26.6% or 0.2676

So Answer is B

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Soln. :**

We have a normal distribution with *μ* = 38 and *σ* = 6

Let X be the number of employees.

1. Probability of employees greater than age of 44

P(X > 44) = 1 - P(X ≤ 44)

Z = (X - 38)/6

P(X ≤ 44) = P(Z ≤ (44 - 38)/6)

= P(Z ≤ 1)=84.1345%

Probability that the employee will be greater than age of 44is 100-84.1345=15.86

So, the probability of number of employees between 38-44 years of age = P(X<44) - 0.5

= 84.1345 - 0.5

= 34.1345%

Therefore the statement that More employees at the processing center are older than 44 than between 38 and 44 is TRUE.

1. Probability of employees less than age of 30 = Pr(X<30).

Z = (30 - 38)/6

P(X ≤ 30) = P(Z ≤ (30 - 38)/6)

= P(Z ≤ -1.333)=9.12%

So, the number of employees with probability 0.912 of them being under age 30

is 0.0912\*400=36.48. Approx. 36 Employees

Therefore the statement B of the question is also TRUE.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Soln. :**

As we know that if *X* ~ N(µ1 , σ12 ) and Y ~ N(µ2, σ22 ) are two independent

random variables

Then X + Y ~ N(µ1 + µ2 , σ12 + σ22 ) , and X − Y ~ N(µ1 − µ2 , σ12 + σ22) .

Similarly, if Z = a*X* + bY , where *X* and Y are as defined above,

i.e Z is linear combination of X and Y , then Z ~ N(aµ1 + bµ2 , a2σ12 + b2σ22 ).

Therefore, X = X1 and Y = *X*2

2X1 ~ N(2 µ,4 σ2) and *X*1 + *X*2 ~ N(µ + µ, σ2 + σ2 ) ~ N(2 µ, 2σ2 )

2X1 - (*X*1 + *X*2) = N( 4µ , 6σ2 )

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Soln. :**

The Probability of getting value between a and b should be 0.99.

The Probability outside the a and b area is 0.01 (i.e. 1-0.99).

The Probability towards left from a = -0.005 (i.e. 0.01/2).

The Probability towards right from b = +0.005 (i.e. 0.01/2).

So, since we have the probabilities of a and b, we need to calculate X, the random

variable at a and b which has got these probabilities.

By finding, the Standard Normal Variable Z (Z Value), we can calculate the X values.

Z=(X- μ) / σ

For Probability 0.005 the Z Value is -2.57

Z \* σ + μ = X

Z(+0.005)\*20+100 = (-2.57)\*20+100 = 48.6

Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4

So, option D is correct.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**Soln. :**

Range is Rs (99.00810347848784, 980.9918965215122) in Millions

1. Specify the 5th percentile of profit (in Rupees) for the company

**Soln. :**

5th percentile of profit (in Million Rupees) is 170.0

1. Which of the two divisions has a larger probability of making a loss in a given year?

**Soln. :**

Probability of Division 1 making a loss P(X<0) = 0.047

Probability of Division 2 making a loss P(X<0) = 0.040